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Thermodynamics of natural convection in enclosures with viscous dissipation

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Abstract

Consideration of the viscous dissipation effects in natural and mixed convection heat transfer must be taken carefully, both in what concerns the thermodynamics of the problem and the relevance of the dissipation term. This applies equally to external or internal natural and mixed convection, and to spaces filled with a single fluid or to spaces filled with fluid-saturated porous media. The main question is related to the fact that, in natural convection, the work done by the pressure forces must equal the energy dissipated by viscous effects, which is the unique situation compatible with the First Law of Thermodynamics, the net energy generation in the overall domain being zero. If only the (positive) viscous dissipation term is considered in the energy conservation equation, the domain behaves like a heat multiplier, the heat output being higher than the heat input. If this is not taken into consideration, erroneous conclusions about flow and temperature fields and heat transfer results are obtained. In mixed convection problems, part of the viscous dissipation term is equally due to the work of pressure forces. Attention is given mainly to the natural convection problem in a square enclosure, the main conclusions applying for general natural or mixed convection heat transfer problems.

1. Introduction

Viscous dissipation is not usually taken into account when dealing with natural convection heat transfer problems. From an order of magnitude analysis it can be concluded that such an approach is usually correct, both when the domains are filled with a single fluid or when the domains are filled with fluid-saturated porous media. Some works can be referred, however, that include the viscous dissipation into the energy equation, for some natural convection problems in open fluid domains [1–6] or in domains filled with a fluid-saturated porous medium [7–11], as well as for mixed convection problems [12–17]. The present work is mainly motivated by some works dealing with natural and mixed convection heat transfer problems taking into account the viscous dissipation effects without considering the work of pressure forces, to which some comments must be addressed and some clarifications need to be made.

Emphasis is given to the natural convection heat transfer problem in a square enclosure heated from the side, with upper and lower perfectly insulated walls, but the main conclusions apply equally to any natural or mixed convection problem. Important conclusions are also obtained for a better understanding of the natural convection heat transfer problem as a heat engine.

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Nomenclature

c_P	constant pressure specific heat
Da	Darcy number
Ec	Eckert number
g	gravitational acceleration
H	height
K	permeability
Nu	Nusselt number
p	pressure
Pr	Prandtl number
$\dot{Q}$	heat flow, by unit depth
Ra	Rayleigh number
$\dot{S}$	entropy flow
T	temperature
u, v	Cartesian velocity components
V	volume
$\dot{W}$	mechanical power, by unit depth
x, y	Cartesian co-ordinates

Greek symbols

α	thermal diffusivity
β	volumetric expansion coefficient
ΔT	temperature difference
ν	kinematic viscosity
ρ	density
ψ	streamfunction

Subscripts

C	cold (lower temperature) value
CD	conduction
D	viscous dissipation
gen	generation
H	hot (higher temperature) value
in	inlet
out	outlet
0	reference value
*	dimensionless

Superscript

d	driving value

From a thermodynamic viewpoint, a lumped (exact) energy analysis can be made to the enclosure, showing that special care must be taken when considering the viscous dissipation. The energy balance for the enclosure requires the heat input equal to the heat output of the enclosure. In this way, the global heat generation in the domain must be null. This is true in both cases, where the viscous dissipation is considered or not. However, when it is considered, an apparent paradox exists: how can the heat output equal the heat input if a positive viscous dissipation exists? The answer is that the term corresponding to the work of pressure forces must also be considered in the energy equation and, when integrated over the overall domain, it equals the integral of the viscous dissipation term over the domain. If the work of pressure forces is not taken into account, if it is not taken into account in the correct way, or if the viscous dissipation term is not taken into account in the correct way, artificial heat generation or destruction can be introduced. If it is the case, erroneous results for the flow and temperature fields and for the heat transfer performance of the enclosure are obtained. The influence of the work of pressure forces on some natural convection heat transfer problems has been the subject of previous works [3,6,18,19].

Also important is the fact that many natural convection studies use the Boussinesq approximation and the dimensionless version of the governing equations. In order to increase the importance of the viscous dissipation, some non-realistic values can be given to the dimensionless governing parameters, thus leading to results associated to non-realistic situations.

2. Thermodynamic analysis

Natural convection heat transfer in enclosures heated from the side can be analyzed from the thermodynamic viewpoint, considering the models present in Fig. 1a and b. In particular, a square enclosure with side length H is considered.

Near the left vertical hot isothermal wall, part of the heat input increases the temperature of the fluid, which expands and rises in level. The remaining heat input is transferred by conduction towards the cold wall. It is assumed that the involved fluid expands when its temperature increases ($\beta > 0$), even if some particular cases can be pointed out for which $\beta < 0$. Near the right vertical cold isothermal wall the heat released results in a decrease of the fluid temperature, the fluid contracts and sinks down. As the enclosure is closed, a loop is established for the fluid flow, and a combined conduction–convection action that promotes heat transfer from the hot wall to the cold wall occurs. The so originated fluid current could move a propeller if it were present. In reality, the viscous dissipation partially brakes the fluid flow, in a way similar to the propeller if it were present in the fluid current. In this way, an equilibrium situation is reached, the power obtained from the expansion–contraction cycle being viscously dissipated as heat. This is

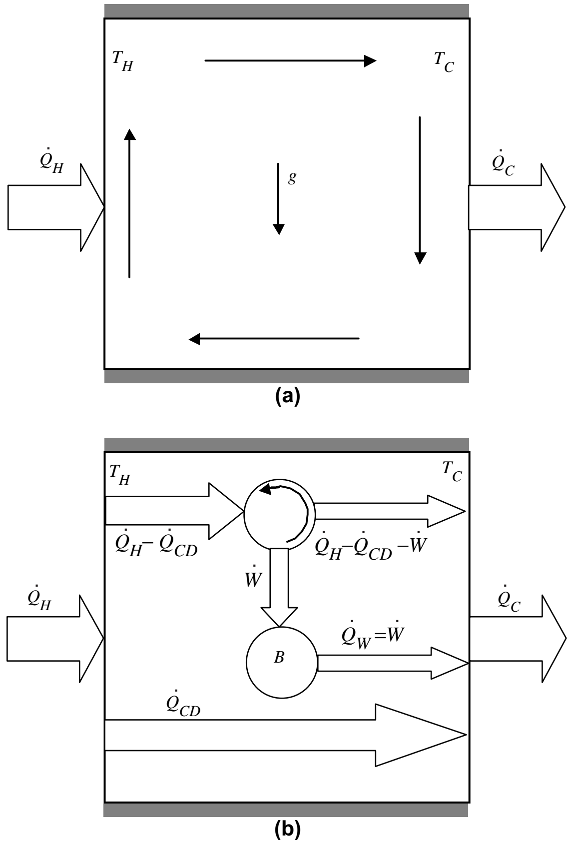

Fig. 1. The natural convection problem in a enclosure heated from the side: (a) usual flow and heat transfer picture; and (b) thermodynamic model including the work of pressure forces and the viscous dissipation.

the picture of the natural convection problem as presented in Fig. 1a.

Its corresponding thermodynamic model is presented in Fig. 1b. The difference of temperature through the enclosure is used to operate a reversible thermal engine, which uses part of the heat input and delivers the mechanical power $\dot{W}$. This mechanical power is integrally dissipated in a brake (the thermodynamic model of the viscous dissipation mechanism), and the dissipated heat is released at the right cold vertical wall. It can be seen that, even when the viscous dissipation is considered,

$$\dot{Q} = |\dot{Q}_C| = |\dot{Q}_H| \tag{1}$$

In fact, the First Law of Thermodynamics applied for the overall enclosure (a closed system) gives $(dE/dt) = \dot{Q} + \dot{W}$, where E is the total energy, $E = U + (1/2)mV^2 + mgz$, and $\dot{Q}$ and $\dot{W}$ are taken as positive when entering the thermodynamic system. If natural convection takes place in steady-state $\dot{Q} + \dot{W} = 0$ or, expanding $\dot{Q}$ in its contributions $\dot{Q}_H$ and $\dot{Q}_C$, $\dot{Q}_H + \dot{Q}_C + \dot{W} = 0$. As there is no any rotating shaft or other mechanical device through which the enclosure exchanges work with its neighborings it is $\dot{W} = 0$, and $\dot{Q}_H\ (>0) + \dot{Q}_C\ (<\ 0) = 0$, that is, $\dot{Q} = |\dot{Q}_C| = |\dot{Q}_H|$. This result is independent of the medium that fills the enclosure.

In that case, the work of pressure forces (mechanical power $\dot{W}$) globally equals the viscous dissipation, $\dot{Q}_W$. In steady-state conditions, no heat is gained or lost in the enclosure, and it is to be noted that flow exists as the result of using some of the heat input. If the viscous dissipation term is taken into account in the energy conservation equation, without considering the work of pressure forces, a net heat generation occurs in the enclosure, and it is $|\dot{Q}_C| = |\dot{Q}_H| + |\dot{Q}_W| > |\dot{Q}_H|$. This is, however, a situation violating the First Law of Thermodynamics, that applied over the overall enclosure gives $|\dot{Q}_C| = |\dot{Q}_H|$. If it is $|\dot{Q}_C| > |\dot{Q}_H|$ the enclosure behaves as a heat multiplier, for which $|\dot{Q}_{out}| > |\dot{Q}_{in}|$. Only if any heat generation of different nature than the viscous dissipation or work of pressure forces, like a chemical reaction or an electrical resistance, it is $|\dot{Q}_C| > |\dot{Q}_H|$.

From the Second Law of Thermodynamics, the rate of entropy generation in the enclosure is

$$\dot{S}_{gen} = (\dot{S}_{gen})_{CD} + (\dot{S}_{gen})_{D} = \dot{Q}\left(\frac{1}{T_C} - \frac{1}{T_H}\right) \tag{2}$$

where $(\dot{S}_{gen})_{CD}$ is the rate of entropy generation due to heat diffusion and $(\dot{S}_{gen})_{D}$ is the rate of entropy generation due to viscous dissipation. Each of these terms can be evaluated alone [20], but the overall rate of entropy generation in the domain can be evaluated from an overall entropy balance for the enclosure, which gives $\dot{S}_{gen} = \dot{Q}(1/T_C - 1/T_H)$.

3. Enclosure filled with a single fluid

Usually, without considering the viscous dissipation term and the work of pressure forces, only the convection–diffusion energy contributions of the problem are considered. In this case, $|\dot{Q}_C| = |\dot{Q}_H|$, and in terms of the overall Nusselt numbers $Nu_C = Nu_H$. By the aforementioned reasons $Nu_C = Nu_H$ must hold to have a consistent energy formulation, with or without considering the complete energy form of the problem.

3.1. Physical modeling

The dimensionless version of the complete equations governing the steady natural convection heat transfer problem, for a fluid of constant properties other than density, are [20–22]

$$\frac{\partial}{\partial x_*}(\rho_* u_*) + \frac{\partial}{\partial y_*}(\rho_* v_*) = 0 \tag{3}$$

$$\frac{\partial}{\partial x_*}(\rho_* u_* u_*) + \frac{\partial}{\partial y_*}(\rho_* v_* u_*) = -\frac{\partial p_*^{\mathrm{d}}}{\partial x_*} + Pr\left(\frac{\partial^2 u_*}{\partial x_*^2} + \frac{\partial^2 u_*}{\partial y_*^2}\right) + \frac{1}{3} Pr \frac{\partial}{\partial x_*}\left(\frac{\partial u_*}{\partial x_*} + \frac{\partial v_*}{\partial y_*}\right) \tag{4}$$

$$\frac{\partial}{\partial x_*}(\rho_* u_* v_*) + \frac{\partial}{\partial y_*}(\rho_* v_* v_*) = -\frac{\partial p_*^{\mathrm{d}}}{\partial y_*} + Pr\left(\frac{\partial^2 v_*}{\partial x_*^2} + \frac{\partial^2 v_*}{\partial y_*^2}\right) + \frac{1}{3} Pr \frac{\partial}{\partial y_*}\left(\frac{\partial u_*}{\partial x_*} + \frac{\partial v_*}{\partial y_*}\right) + Ra\, Pr\, \beta_* T_* \tag{5}$$

$$\frac{\partial}{\partial x_*}(\rho_* u_* T_*) + \frac{\partial}{\partial y_*}(\rho_* v_* T_*) = \frac{\partial^2 T_*}{\partial x_*^2} + \frac{\partial^2 T_*}{\partial y_*^2} + Ec\, \beta_0 T_0 \beta_* \left(\frac{\Delta T}{T_0} T_* + 1\right) \times \left[u_* \frac{\partial p_*^{\mathrm{d}}}{\partial x_*} + v_* \left(\frac{\partial p_*^{\mathrm{d}}}{\partial y_*} - \frac{Ra\, Pr}{\beta_0 \Delta T}\right)\right] + Ec\, Pr\, \Phi_* \tag{6}$$

where the dimensionless dissipation function is given by

$$\Phi_* = 2\left[\left(\frac{\partial u_*}{\partial x_*}\right)^2 + \left(\frac{\partial v_*}{\partial y_*}\right)^2\right] + \left(\frac{\partial u_*}{\partial y_*} + \frac{\partial v_*}{\partial x_*}\right)^2 - \frac{2}{3}\left(\frac{\partial u_*}{\partial x_*} + \frac{\partial v_*}{\partial y_*}\right)^2 \tag{7}$$

and the variables were made dimensionless as

$$(x_*, y_*) = (x, y)/H \tag{8a}$$

$$(u_*, v_*) = (u, v)/(\alpha/H) \tag{8b}$$

$$p_*^{\mathrm{d}} = (p + \rho_0 g y)/[\rho_0 (\alpha/H)^2] \tag{8c}$$

$$\rho_* = \rho/\rho_0 \tag{8d}$$

$$T_* = (T - T_{\mathrm{C}})/\Delta T \tag{8e}$$

$$\beta_* = \beta/\beta_0 \tag{8f}$$

with $\Delta T = (T_{\mathrm{H}} - T_{\mathrm{C}})$ and $T_0 = T_{\mathrm{C}}$ is in the absolute scale. In this way, the dimensional absolute temperature is obtained from the dimensionless temperature T_* as $T = T_0[(\Delta T/T_0)T_* + 1]$. It is assumed that density depends on temperature through the relation $\rho = \rho_0[1 - \beta(T - T_0)]$. The emerging dimensionless governing parameters are

$$Pr = \nu/\alpha \tag{9a}$$

$$Ra = \frac{g\beta_0 \Delta T H^3}{\nu\alpha} \tag{9b}$$

$$Ec = \frac{(\alpha/H)^2}{c_P \Delta T} \tag{9c}$$

Sometimes the Gebhart number is used, defined as $Ge = g\beta_0 H/c_P$, being $Ec\, Pr = Ge/Ra$.

If the fluid has constant density, or if the Boussinesq approximation is considered, it is $\rho_* = 1$, $\beta = \beta_0$ and $\beta_* = 1$, and the term $(\partial u_*/\partial x_* + \partial v_*/\partial y_*)$ vanishes in Eqs. (4), (5) and (7). It is assumed here that the Boussinesq approximation is introduced once the energy conservation equation obtained, thus retaining the term corresponding to the work of pressure forces. If the fluid is taken as an ideal gas, $\beta = 1/T$ and $\beta_* = T_0/T = 1/[(\Delta T/T_0)T_* + 1]$, and it is $\beta_0 T_0 \beta_*[(\Delta T/T_0)T_* + 1] = 1$ in Eq. (6).

Natural convection heat transfer problem is usually solved in its dimensionless form, taking the dimensionless driving pressure p_*^{d} as defined by Eq. (8c), which is the unique pressure force related with fluid motion. However, pressure entering into the energy conservation equation, Eq. (6), is the (thermodynamic) absolute pressure, as thermodynamic relations have been used to obtain the expression for the work of pressure forces [21,22], and it is $(\partial p_*/\partial x_*) = (\partial p_*^{\mathrm{d}}/\partial x_*)$ and $(\partial p_*/\partial y_*) = (\partial p_*^{\mathrm{d}}/\partial y_*) - Ra\, Pr/(\beta_0 \Delta T)$. Nonisotropic normal stresses can exist in a fluid in motion, and only the assumption of local equilibrium allows the mean compressive stress to be interpreted as the thermodynamic pressure. In particular, special care needs to be taken when interpreting pressure for incompressible fluids in motion [22].

Looking on Eq. (6), the two last terms can be identified as the work of pressure forces and the viscous dissipation, respectively. By the reasons explained above, when dealing with thermodynamic analysis, application of the First Law of Thermodynamics to the overall enclosure gives

$$\frac{(-\dot{Q}_{\mathrm{H}} - \dot{Q}_{\mathrm{C}})}{k\Delta T} + \int_{V_*} Ec\, \beta_0 T_0 \beta_* \left(\frac{\Delta T}{T_0} T_* + 1\right) \times \left[u_* \frac{\partial p_*^{\mathrm{d}}}{\partial x_*} + v_* \left(\frac{\partial p_*^{\mathrm{d}}}{\partial y_*} - \frac{Ra\, Pr}{\beta_0 \Delta T}\right)\right] \mathrm{d}V_* + \int_{V_*} Ec\, Pr\, \Phi_* \, \mathrm{d}V_* = 0 \tag{10}$$

where the integrals extend to the overall domain of the enclosure. The volume integral of the divergence of the dimensionless heat flux was transformed into surface integrals using the Gauss' Theorem, and they give the dimensionless heat flows (the Nusselt numbers) entering and leaving the domain. As $\dot{Q}_{\mathrm{H}} + \dot{Q}_{\mathrm{C}} = 0$, it is

$$\int_{V_*} Ec\, \beta_0 T_0 \beta_* \left(\frac{\Delta T}{T_0} T_* + 1\right) \times \left[u_* \frac{\partial p_*^{\mathrm{d}}}{\partial x_*} + v_* \left(\frac{\partial p_*^{\mathrm{d}}}{\partial y_*} - \frac{Ra\, Pr}{\beta_0 \Delta T}\right)\right] \mathrm{d}V_* + \int_{V_*} Ec\, Pr\, \Phi_* \, \mathrm{d}V_* = 0 \tag{11}$$

Locally, the term corresponding to the work of pressure forces and the viscous dissipation term can be different. However, the integral of these two energy terms extended to the overall enclosure must cancel.

Thus, in order to have heat transfer results consistent with the First Law of Thermodynamics, special care must be taken when considering the viscous dissipation effects in the energy conservation equation, noting that: (i) Result $|\dot{Q}_C| = |\dot{Q}_H|$ or, equivalently, $Nu_C = Nu_H$, is the unique relation respecting the First Law of Thermodynamics; (ii) not only the overall values of the Nusselt numbers must satisfy $Nu_C = Nu_H$, but the fluid and temperature fields must be evaluated from the correct formulation of the energy conservation equation, including all the relevant sources and sinks; (iii) integration methods used must be consistent with the result given by Eq. (11); (iv) pressure involved in the work of pressure term is the (total) absolute pressure and not the driving pressure; and (v) assessment of using the Boussinesq approximation must be related not only with the difference on the involved thermal levels [23], but also on the verification of Eq. (11), noting that the influence of the density variations spread over the involved equations.

3.2. Numerical modeling and analysis of results

The natural convection problem in the square enclosure with side length H was solved in its dimensionless form using a control volume finite element method [24], with a 75×75 non-uniform mesh, which expands from the walls to the center with a geometric expansion factor equal to 1.05. Results were obtained for the different conditions as detailed in Table 1, and for $Pr = 1$, $Ra = 10^5$, $\Delta T = 10$ K, and $\beta_0 = 1/T_0$ K^{-1}. In this work, ideal gas means only that $\beta T = 1$, and not that a closed thermodynamic relation exists between temperature and pressure. In other words, the fluid is taken as incompressible but dilatable. For any fluid, temperature is obtained from the energy conservation equation and pressure is obtained from the flow solution.

From the results of Table 1 it is clearly concluded that erroneous results are obtained if only the viscous dissipation term is considered into the energy conservation equation, in what concerns the flow and temperature fields, and also the overall Nusselt number of the enclosure. When the viscous dissipation and the work of pressure forces are taken into account, the small differences on the numerical values of the Nusselt numbers and on the integrals of such terms can be attributed to the numerical approximations introduced to obtain them. Even so, the obtained results are coherent. For example, introduction of results from the fourth line of Table 1 into Eq. (10) lead to $(-13.258 + 13.139) + (0.1191 - 0.2384) = 0$, that is, $0.119 - 0.119 = 0$. It is also observed that the overall Nusselt number is not significantly affected by the consideration of the Boussinesq approximation or the ideal gas consideration ($\beta T = 1$), once the remaining conditions are fixed.

For comparison purposes, the streamlines and the isotherms for the situations corresponding to the first and second rows of Table 1 are presented in Fig. 2a and b, respectively. It is clear that an inconsistent formulation of the energy conservation (Fig. 2a and first row of Table 1) leads to considerably different results when compared with the ones corresponding to the consistent formulation of the energy conservation (Fig. 2b and second row of Table 1). In isotherms of Fig. 2a it is observed that thy are denser near the vertical right cold wall, thus indicating that $|\dot{Q}_C| > |\dot{Q}_H|$. When the work of pressure forces is taken into account, as presented in Fig. 2b, both the flow and temperature fields present significant changes. In this case, flow occurs only very close to the walls, and the isotherms are very dense close to the walls and very sparse in the interior of the enclosure. Thus, intense temperature gradients exist close to the walls, leading to higher values of the Nusselt number, in this case the same overall value at the left and right vertical walls. As present in Table 1, very different results are obtained for the Nusselt numbers when the work of pressure forces is considered or not, and those

Table 1
Different conditions and terms in the energy conservation equation, Nusselt numbers at the hot and cold vertical walls, and results for the dimensionless overall viscous dissipation, D_* and dimensionless overall work of pressure forces, W_*

Ec	T_0 [K]	Boussinesq approximation	Ideal gas	Viscous dissipation	Work of pressure forces	Nu_H	Nu_C	D_*	W_*
10^{-5}	300	Yes	No	Yes	No	3.658	5.565	1.907	0.000
10^{-5}	300	Yes	No	Yes	Yes	13.198	13.200	0.1192	−0.1172
10^{-5}	300	No	No	Yes	No	3.621	5.551	1.931	0.000
10^{-5}	300	No	No	Yes	Yes	13.258	13.139	0.1191	−0.2384
10^{-5}	300	No	Yes	Yes	No	3.617	5.465	1.848	0.000
10^{-5}	300	No	Yes	Yes	Yes	13.024	13.024	0.1157	−0.1157
10^{-6}	300	No	Yes	Yes	Yes	7.053	7.052	0.06096	−0.0613
10^{-7}	300	No	Yes	Yes	Yes	4.884	4.883	0.01582	−0.01617
10^{-5}	500	No	Yes	Yes	Yes	15.028	15.028	0.08008	−0.08005

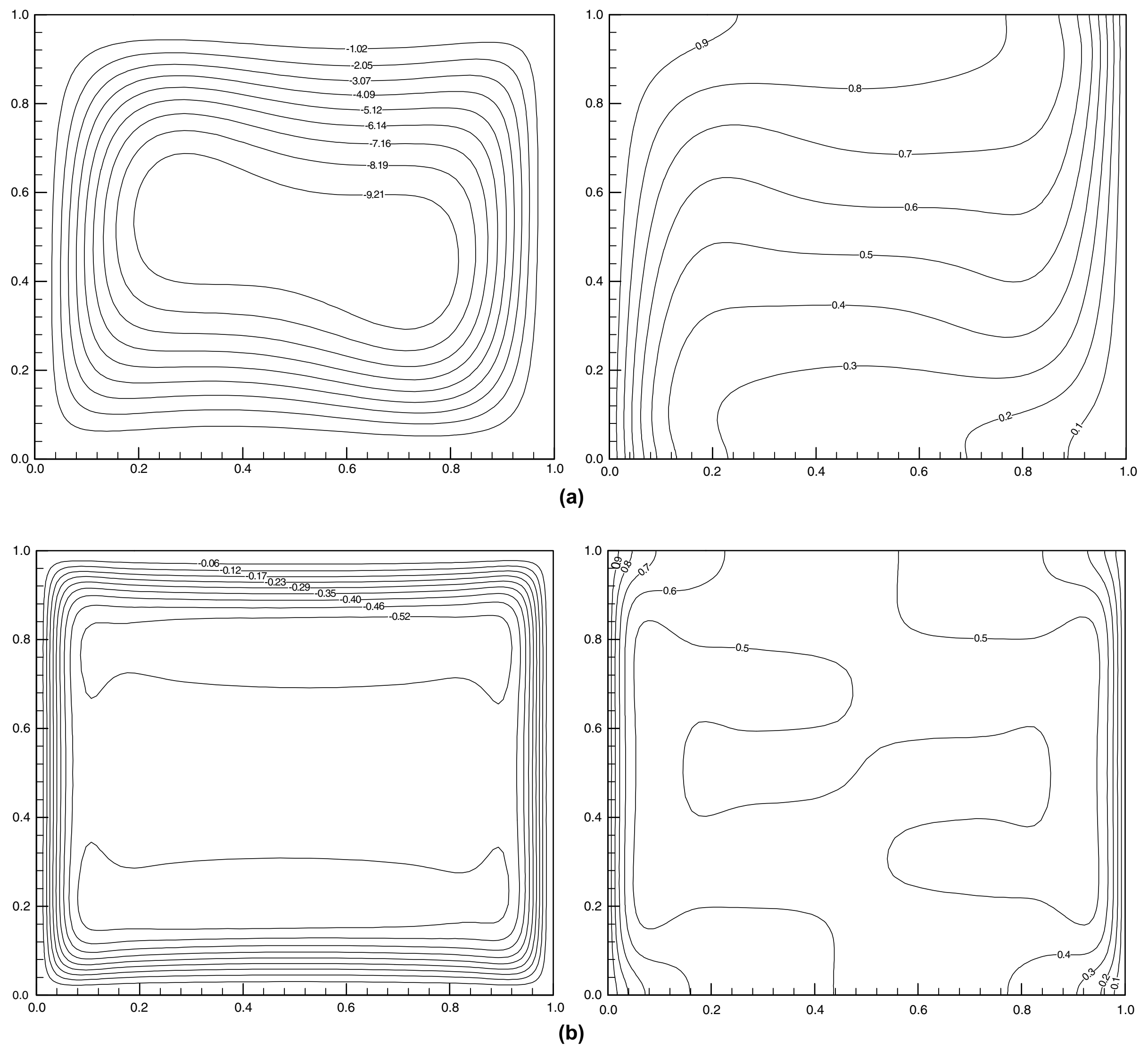

Fig. 2. Comparison of the flow (left) and temperature (right) fields: (a) without considering the work of pressure forces (conditions of the first row of Table 1); and (b) considering the work of pressure forces (conditions of the second row of Table 1).

corresponding to the first row are inconsistent regarding the First Law of Thermodynamics. As it is $(\partial p_*/\partial x_*) = (\partial p_*^{\mathrm{d}}/\partial x_*)$ and $(\partial p_*/\partial y_*) = (\partial p_*^{\mathrm{d}}/\partial y_*) - Ra\,Pr/(\beta_0 \Delta T)$, if the vertical pressure gradient is dominated by the term $-Ra\,Pr/(\beta_0 \Delta T) \approx -Ra\,Pr$ it is $v_*(\partial p_*/\partial y_*) < 0$ near the left vertical wall and $v_*(\partial p_*/\partial y_*) > 0$ near the right vertical wall. Thus, taking present the energy conservation equation, Eq. (6), the work of pressure forces acts like a heat sink close to the left hot vertical wall and like a heat source close to the right cold vertical wall. This is the main reason for the high values of the Nusselt number obtained when the work of pressure forces is taken into account.

3.3. The natural convection problem in enclosures as a heat engine

For a better understanding of the natural convection problem as a heat engine, as presented in Fig. 1b and explained in Section 2, the best way is to analyze the entropy generation equation, Eq. (2). From that equation, and from [20], it can be stated that

$$\dot{S}_{\mathrm{gen}} = (\dot{S}_{\mathrm{gen}})_{\mathrm{CD}} + (\dot{S}_{\mathrm{gen}})_D = \int_V \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]\mathrm{d}V + \int_V \frac{\mu}{T}\Phi\,\mathrm{d}V \quad (12)$$

whose dimensionless version is

$$\dot{S}_{gen,*} = \frac{\dot{S}_{gen} T_0}{k\Delta T} = \underbrace{\int_{V_*} \frac{\Delta T}{T_0} \frac{1}{[(\Delta T/T_0)T_* + 1]^2} \left[\left(\frac{\partial T_*}{\partial x_*} \right)^2 + \left(\frac{\partial T_*}{\partial y_*} \right)^2 \right] dV_*}_{(\dot{S}_{gen,*})_{CD}} + \underbrace{\int_{V_*} \frac{Ec\,Pr\,\Phi_*}{[(\Delta T/T_0)T_* + 1]} dV_*}_{(\dot{S}_{gen,*})_{D}} \tag{13}$$

For example, for the situation in the last row of Table 1, it is $(\dot{S}_{gen,*})_{CD} = 0.07930$ and $(\dot{S}_{gen,*})_{D} = 0.2152$, and it is $(\dot{S}_{gen,*})_{CD} + (\dot{S}_{gen,*})_{D} = 0.2945$. From the dimensionless version of Eq. (2) it can be obtained that $(\dot{S}_{gen,*}) = Nu(\Delta T/T_0)(\Delta T/T_0 + 1)^{-1} = 0.2947$.

In this way, the natural convection heat transfer problem can be interpreted as schematically presented in Fig. 1b, where part of the entering heat is transferred by conduction, and the remaining is used to feed the 'heat engine' that causes the fluid motion, the work delivered by such an engine being viscously dissipated as heat. This quantitative analysis completes the description of the natural convection problem as a heat engine, as explained in [20], and it gives a more precise analysis of the problem when compared with the study reported in [25].

Another possible model is to consider heat being entirely transferred by conduction between temperature T_H and an intermediate temperature $T'_H < T_H$, as well as between a lower intermediate temperature $T'_C > T_C$ and the lowest temperature T_C. Between temperatures T'_H and T'_C heat is used to drive a thermal engine, which delivers mechanical power that is entirely dissipated by friction. This corresponds to the heat engine and the brake system operating between the intermediate temperature levels T'_H and T'_C only.

The illustration of the lost available work process as a brake dissipating the mechanical power delivered by a heat engine operating between two different thermal levels has been proposed by Bejan [26,27], when analyzing the irreversibility associated with the heat transfer process across a nonzero temperature difference. This same model has been used by Bejan [28], to explain the steady natural convection heat transfer process as the situation where the useful delivered mechanical power, in the expansion–contraction cycle experienced by the operating fluid, is entirely dissipated by friction.

3.4. Numerical values of the dimensionless governing parameters

The natural convection problem is usually solved in its dimensionless form, whose results must refer to practical situations. In what concerns the numerical values assumed by the dimensionless governing parameters, care must be taken about the values given to such parameters and their relevance in dimensional terms. For air, water or other common fluids at room temperature, it can be seen that the practical laminar situations lead to $Ra \sim 10^3$–10^9 and $Ec < \sim 10^{-9}$, parameters for which the dissipation effects are not 'visible' on the flow and temperature fields and on the heat transfer parameters. In an attempt to increase the importance of the viscous dissipation effects, it is tempting to increase Ec. However, such values of the governing dimensionless parameters can make the viscous dissipation 'visible' on the flow and temperature fields and on the heat transfer parameters but they do not correspond to realistic situations.

4. Enclosures filled with a fluid-saturated porous medium

The main aspects referred above for enclosures filled with a single fluid apply equally to enclosures filled with a fluid-saturated porous medium. The flow within the porous medium can be modeled using the Darcy Law (for low fluid velocities) or using more elaborate models like the Brinkman–Forchheimer flow model [29]. In what follows the Darcy flow model is used, the main conclusions remaining unchanged even if more elaborate models are used. Following Nield and Bejan [29], the viscous dissipation term corresponding to the use of the Brinkman–Forchheimer model is close to the one corresponding to the simpler Darcy flow model, as given by the last term in Eq. (16). Also, in this case, care should be taken when considering the applicability of the Boussinesq approximation.

The velocity components and pressure are linked through the Darcy Law,

$$u_* = -\frac{Da}{Pr} \frac{\partial p_*}{\partial x_*} \tag{14a}$$

$$v_* = -\frac{Da}{Pr} \frac{\partial p_*}{\partial y_*} + Ra\,T_* - \frac{Ra}{\beta_0 \Delta T} \tag{14b}$$

and the complete equations governing the natural convection heat transfer problem, for a Boussinesq fluid ($\beta_* = 1$), are

$$0 = \frac{\partial^2 \psi_*}{\partial x_*^2} + \frac{\partial^2 \psi_*}{\partial y_*^2} + Ra \frac{\partial T_*}{\partial x_*} \tag{15}$$

$$\frac{\partial}{\partial x_*}(\rho_* u_* T_*) + \frac{\partial}{\partial y_*}(\rho_* v_* T_*) = \frac{\partial^2 T_*}{\partial x_*^2} + \frac{\partial^2 T_*}{\partial y_*^2} + Ec\,\beta_0 T_0 \left(\frac{\Delta T}{T_0} T_* + 1 \right) \left(u_* \frac{\partial p_*}{\partial x_*} + v_* \frac{\partial p_*}{\partial y_*} \right) + \frac{Ec\,Pr}{Da}(u_*^2 + v_*^2) \tag{16}$$

where the dimensionless variables and governing parameters were defined as

$$p_* = p/[\rho_0(\alpha/H)^2] \quad (17a)$$

$$\psi_* = \psi/\alpha \quad (17b)$$

$$Ra = \frac{g\beta_0 \Delta T K H}{\nu\alpha} \quad (17c)$$

$$Da = \frac{K}{H^2} \quad (17d)$$

In the foregoing equations, the terms corresponding to the work of pressure forces and to the viscous dissipation were modeled as suggested by Nield and Bejan [29]. The velocity and pressure are linked though the Darcy Law, and the components of the pressure gradient to introduce in Eq. (16) can be obtained as

$$\frac{\partial p_*}{\partial x_*} = -\frac{Pr}{Da}u_* \quad (18a)$$

$$\frac{\partial p_*}{\partial y_*} = -\frac{Pr}{Da}v_* + \frac{Ra\,Pr}{Da}T_* - \frac{Ra\,Pr}{Da}\frac{1}{\beta_0\Delta T} \quad (18b)$$

Also in this case, and by the same reasons as explained above, for the enclosures filled with a single fluid, it must be

$$\int_{V_*} Ec\,\beta_0 T_0\left(\frac{\Delta T}{T_0}T_* + 1\right)\left(u_*\frac{\partial p_*}{\partial x_*} + v_*\frac{\partial p_*}{\partial y_*}\right)\mathrm{d}V_* + \int_{V_*}\frac{Ec\,Pr}{Da}(u_*^2 + v_*^2)\,\mathrm{d}V_* = 0 \quad (19)$$

where the volume integrals extend to the overall enclosure. The local terms corresponding to the work of pressure forces and to the viscous dissipation term can be different. However, Eq. (19) must be verified to have heat transfer results consistent with the First Law of Thermodynamics.

If the problem is solved in its dimensionless form, care must be taken in what concerns the numerical values assumed by the dimensionless governing parameters. For air, water or other common fluids at room temperature, it can be seen that the practical laminar situations lead to $Ra \sim 10$–10^3 and $Ec < \sim 10^{-9}$, parameters for which the dissipation effects are not 'visible'. In an attempt to increase the importance of the viscous dissipation effects, it is tempting to increase Ec, to levels that do not correspond to realistic situations [10,11]. From Eqs. (9c), (17b) and (17c) it can be obtained that

$$H = Ra\left(\frac{Ec\,Pr}{Da}\right)\left(\frac{c_P}{g\beta_0}\right) \quad (20)$$

Taking $Ra = 10^3$ and $(Ec\,Pr/Da) = 10^{-3}$, for fluids at room temperature it is: $H = 3.1 \times 10^4$ m for air, $H = 1.5 \times 10^6$ m for water, and $H = 2.8 \times 10^5$ m for oil, that is, non-realistic values for the side length of the differentially heated square enclosure.

5. Conclusions

Convection heat transfer problems including the effect of viscous dissipation should be carefully modeled in order to respect the First Law of Thermodynamics. For this problem, both the viscous dissipation term and the work of pressure forces must be included into the energy conservation equation, due to the fact that is the work of pressure forces that moves the fluid, and this energy action is compensated by the viscous dissipation effect. If it is not the case, such enclosures behave as heat generators, violating the First Law of Thermodynamics. This applies equally to enclosures filled with a single fluid or to enclosures filled with fluid-saturated porous media.

Recognizing that the integral of the work of the pressure forces must be equal to the integral of the viscous dissipation, the integrals extending to the overall enclosure, one has a very useful criterion that can be used in order to assess how correct are the models used to account for the work of the pressure forces and for the viscous dissipation. When dealing with enclosures filled with a single fluid the model is established in an 'exact' way, but the usual use of the Boussinesq approximation must be assessed taking into account both the maximum temperature difference and the equality of such integrals. When dealing with enclosures filled with fluid-saturated porous media, the model is established on a comparative basis, taking as reference what happens in the enclosures filled with a single fluid. Also in that case the use of the Boussinesq approximation must be assessed taking into account both the maximum temperature difference and the equality of such integrals.

From a careful analysis of the simple problem of natural convection heat transfer in a square enclosure, important conclusions are extracted which apply also when considering the viscous dissipation effects in other heat transfer problems, involving natural convection heat transfer or mixed convection heat transfer. When dealing with mixed convection heat transfer problems, part of the viscous dissipation term comes from the 'forced' mechanical energy input and part comes from the work of pressure forces (associated with the natural convection component of the flow). Only the consideration of the complete energy conservation equation can give the correct results in what concerns the flow and temperature fields, and the heat transfer parameters.

Also an improved picture is given to see the natural convection heat transfer problem in enclosures as a heat engine.

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